Engineering Notes

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Compressor Distortion Estimates Using Parallel Compressor Theory and Stall Delay

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Introduction

THE parallel compressor model is a concise, reliable method for preliminary estimates of circumferential distortion effects on performance. This was demonstrated by the work of Doyle and Horlock.[1] The method was extended by C. A. Reid[2] to provide prediction of the stability limit degradation caused by circumferential total pressure distortion. Reid's work pointed out that stability limit prediction was dependent on circumferential distortion extent. Early data correlation efforts by the author showed that each compressor type exhibited different, but consistent, degradation in stability limit if a constant value of distortion extent was utilized in the parallel model calculation. It was concluded that an additional effect was influencing stall susceptibility under circumferentially distorted inlet conditions. Rotor stall delay was found to be a major part of this effect and significantly improved the correlation of compressor stall susceptibility with distortion extent.

Parallel Compressor Model

The parallel compressor model assumes that no circumferential flow migration takes place throughout the compressor between the distorted and undistorted sectors. It is also assumed that the receiver static pressure is uniform and that all sectors of the compressor operate on an undistorted characteristic. With these assumptions, a compressor may be sectored into two compressors having the same over-all characteristics but with different inlet total pressures (representing distorted flow) and their weighted sum will approximate one compressor with a distorted inlet total pressure. To estimate changes in the stability limit the calculation assumes that the summed compressor stalls when the distorted compressor sector operates on the clean inlet stall line. This mode is referred to as a unit sensitivity condition and the degradation in stall limit so calculated is used as a reference for correlation of circumferential distortion test data.

Representative results for a unit sensitivity calculation are shown in Fig. 1 with clean inlet and circumferential distortion test data for comparison. The parallel compressor model calculation is conservative in that it estimates a larger stall limit degradation than that realized at test. A consistent method to analytically account for this difference will now be described.

Presented as Paper 74-233 at the AIAA 12th Aerospace Sciences Meeting, Washington, D.C., January 30–February 1, 1974; submitted February 7, 1974; revision received May 3, 1974.

Index categories: Airbreathing Propulsion, Subsonic and Supersonic; Nonsteady Aerodynamics.

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Rotor Stall Delay

The presence of circumferential distortion provides that rotor blade incidence is dependent on its peripheral position. As the blade passes into the distorted sector it sees, ideally, a discontinuous increase in incidence. If the rotor is operating near stall this can produce an incidence higher than the steady-state stall value. Over any number of rotor revolutions with a steady circumferential distortion this abrupt change in incidence is periodic. In summary, the rotor blade incidence is varying about the stall limit point of operation at a disturbance frequency related to the distortion sector width and rotor angular speed.

Work accomplished by many investigators[4-10] has verified that lift response to a sudden change in incidence is delayed and a rotor may experience an incidence change beyond stall without stalling—provided that the change in incidence persists only a short time.

It is reasonable to expect that stall delay of a rotor subjected to circumferential distortion is a major factor contributing to the disagreement between parallel compressor theory estimates of stall limit degradation and test data. If this supposition is correct, then the amount of disagreement should correlate with the reduced frequency of the disturbance caused by the distortion.

Correlation of Concept with Test Data

The parallel compressor theory was applied to data from several single stage compressors and one multistage compressor to provide a unit sensitivity estimate of stall line degradation with circumferential distortion. The error between test data and unit sensitivity calculation was expressed as:

$$S\Theta = \frac{R_c/W_{\text{CLEAN}} - R_c/W_{\text{DISTORTED TEST}}}{R_c/W_{\text{CLEAN}} - R_c/W_{\text{PARALLEL MODEL}}} = \frac{\Delta(R_c/W)_{\text{TEST}}}{\Delta(R_c/W)_{2C11}}$$

where R_c is pressure ratio and W is inlet corrected airflow, all values taken at the same corrected speed for the respective stall limit condition. The factor $S\theta$ represents

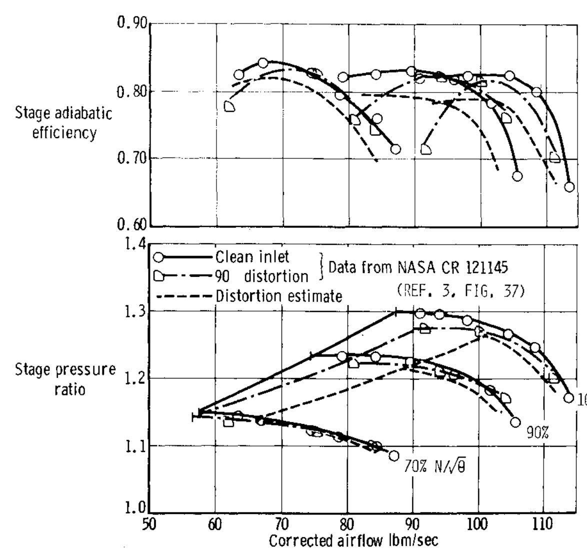

Fig. 1 **Parallel compressor model calculation of circumferential distortion effects with unit sensitivity.**

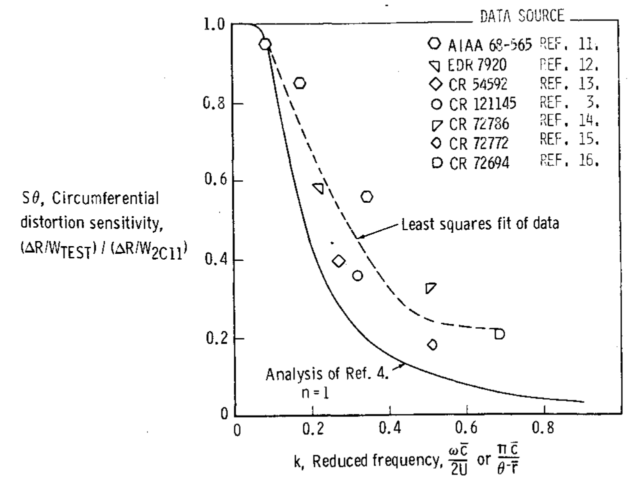

Fig. 2 Correlation of circumferential distortion sensitivity with reduced frequency.

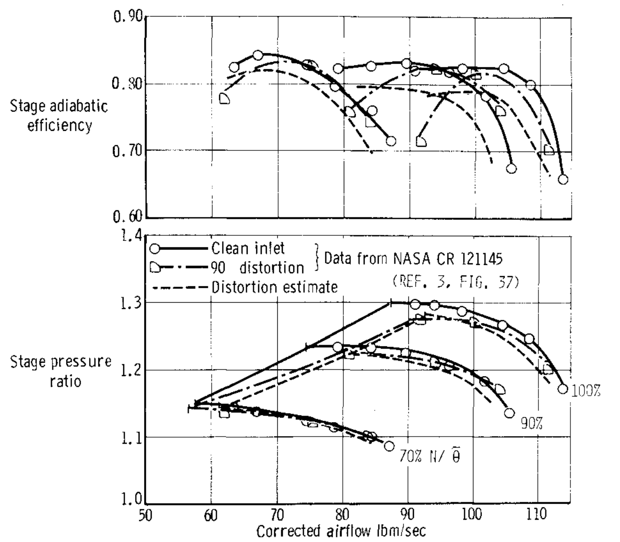

Fig. 3 Representative results of parallel compressor model prediction incorporating sensitivity related to stall delay.

compressor sensitivity to circumferential distortion. Reduced frequency was calculated for each test point by assuming that the disturbance frequency was expressed:

$$\omega = (2\pi/\Theta^{-})(d\phi/dt)$$

where θ^{-} is the distorted sector width, and $(d\phi/dt)$ is rotor angular speed.

Using the disturbance frequency ω, a reduced or characteristic frequency was then calculated:

$$k = (\omega c/2U)$$

where

$$U = r(d\phi/dt) \qquad r = \text{radius} \qquad c = \text{blade chord}$$

then

$$k = (\pi c/r\Theta^{-}) \equiv (\omega c/2U)$$

The graph in Fig. 2 shows the results of plotting $S\theta$ vs reduced frequency. Also shown is the analysis for normalized loss in stall margin for a square wave disturbance.[4] Normalized stall margin is defined in the reference as the ratio of an analytically estimated dynamic stall margin to steady-state stall margin and is conceptually identical to the sensitivity parameter, $S\theta$. Data scatter is primarily caused by the inherent inconsistency of stall limit definition of the various test vehicles and facilities and has the largest effect at low sensitivity. Considering this factor, the correlation is good.

Results of the parallel model calculation using a value of $S\theta$ from the least-squares data fit are shown in Fig. 3. Note that the calculation of the effect of distortion on the stall limit has significantly improved from the unit sensitivity calculation shown in Fig. 1.

Conclusions and Recommendations

Similar agreement with circumferential distortion test data has been accomplished on every set of single stage data available. However, this degree of success has not been consistently obtained on multistage compressor data. Reasons for difficulty with correlation of multistage data are the lack of specific identification of the stalling stage, circumferential flow migration in the many blade-free spaces which somewhat invalidates the parallel compressor model and the distinct possibility of a stator-induced stall limit at some operating conditions.

The good correlation obtained with single stage data is reason to recommend that research efforts be concentrated on these areas to provide for application of the stall-delay concept to any axial flow compressor.

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Fuel Optimality of Cruise

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Introduction

IN the paper by Schultz and Zagalsky,[1] the solution characteristics for the minimum fuel-fixed range problem are determined for a number of different mathematical models of aircraft dynamics. Different results are obtained for different sets of equations. The energy state equations are shown to not allow a partial throttle, constant velocity, cruise solution; but because the velocity set is not convex, to have a "chattering" solution which provides the best performance but which may not be realizable with piecewise continuous maximum values of the controls. The equation set with throttle and flight path angle as controls was shown to allow a partial throttle cruise solution. This conclusion was shown to be invalid by Speyer[3] by application of the Generalized Legendre-Clebsch condition.

The following analysis shows that for a higher order set of equations which has lift and thrust as controls, the necessary condition for optimization including the Generalized Legendre-Clebsch condition are satisfied at the cruise point so that the partial throttle cruise condition is a candidate solution for the minimum fuel-fixed range problem.

Problem Formulation

The problem is to find the aircraft trajectory from an initial velocity, altitude, and range to a final velocity, altitude and range using minimum fuel. Mathematically the problem is to minimize the fuel used G,

$$G = \int_{t_o}^{t_f} \sigma T dt \tag{1}$$

Subject to the constraints:

$$\begin{aligned} \overset{\circ}{E} &= (T - D)V/M;\ E(t_o) = E_o;\ E(t_f) = E_f \\ \overset{\circ}{\gamma} &= (L - W)/MV \\ \overset{\circ}{h} &= V \sin\gamma;\ h(t_o) = h_o;\ h(t_f) = h_f \\ \overset{\circ}{x} &= V \cos\gamma;\ x(t_o) = x_o;\ x(t_f) = x_f \end{aligned} \tag{2}$$

where $E = (V^2/2) + gh$ is the specific energy, γ is the flight path angle, x is the range, σ is the specific fuel consumption assumed to be constant, M is the mass, V is the velocity, x is the range, and D is the drag. Drag is given by

$$D = QSC_{d_0}(M) + (KL^2/QS) \tag{3}$$

The control variables are thrust (T) and Lift (L).

Received April 1, 1974.
Index category: Aircraft Performance.
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Problem Solution

The solution to the stated problem can be found by applying the maximum principle which states: The controls are determined from

$$\min_{u \in U} H \tag{4}$$

where

$$H = \sigma T + \lambda_1(T - D)V/M + \lambda_2(L - W)/MV^2 + \lambda_3 V \sin\gamma + \lambda_4 V \cos\gamma$$

$$\overset{\circ}{\lambda}_1 = -\frac{\partial H}{\partial E},\quad \overset{\circ}{\lambda}_2 = -\frac{\partial H}{\partial \gamma},\quad \overset{\circ}{\lambda}_3 = -\frac{\partial H}{\partial h},\quad \overset{\circ}{\lambda}_4 = -\frac{\partial H}{\partial x} \tag{5}$$

and

$$\frac{\partial H}{\partial t} + \frac{dH}{dt} = 0 \qquad H dt_f - \lambda^T dx_f = 0 \tag{6}$$

Robbins[2] has shown that the solution must also satisfy the Generalized Legendre-Clebsch condition which apply to nonsingular and singular arcs. The Generalized Legendre-Clebsch condition is

$$\frac{\partial}{\partial u}\frac{d^q}{dt^q}\left(\frac{\partial H}{\partial u}\right) = 0 \quad \text{for all } t \text{ on the singular arc and } q \text{ odd}$$

$$(-1)^p \frac{\partial}{\partial u}\left[\frac{d^{2p}}{dt^{2p}}\frac{\partial H}{\partial u}\right] \geq 0 \quad \text{for all } t \text{ on the singular arc} \qquad m = 2p \tag{7}$$

According to Robbins, m is the first value where the derivatives of $(d^m/dt^m)(\partial H/\partial u)$ do not vanish identically; i.e., controls can be determined from this condition

$$\frac{d^m}{dt^m}\left(\frac{\partial H}{\partial u}\right) = W_m(x, \lambda, t) + Q_m(x, \lambda, t)u \tag{8}$$

For multiple control variables, the arc may be singular to degree m $w{\cdot}r{\cdot}t$ some of the control variables and to some higher degree with respect to other control variables. By use of Eq. (8), r, of the control variables can be expressed as a function of x, λ, and t and the remaining ("more singular") control variables.

Substitution into the Hamiltonian gives a new Hamiltonian with fewer control variables. The new Hamiltonian and its first m-1 derivatives vanish identically. The mth derivative does not involve control variables but some higher degree derivatives will (in general). This gives a relation like Eq. (8) but with a larger m and a new matrix Q_m of smaller size. If the matrix is nonsingular, the process terminates.

Test of Cruise Solution

We now test the cruise solution:

$$L = W;\ T = D \qquad V = \text{const.};\ h = \text{const.} \tag{9}$$

to determine if the Maximal Principle and the Legendre-Clebsch condition are satisfied. Taking the first partial of H $w{\cdot}r{\cdot}t$ the controls:

$$(\partial H/\partial T) = \sigma + \lambda_1 V/M \qquad \frac{\partial H}{\partial L} = -\lambda_1\left(\frac{\partial D}{\partial L}\right)\frac{V}{M} + \frac{\lambda_2}{MV} \tag{10}$$